SVKM's NMIMS
MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING/
SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING

Programme: B.Tech/ MBA Tech (Computer) Year: II Semester: III

Academic Year: 2019-20

Subject: Mathematics -III

Date: 05 November 2019

Marks: 100
Time: 2.00 pm - 5.00 pm
Durations: 3 (Hrs)
No. of Pages: 08

Final Examination (2019-20)

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover of the Answer Book, which is provided for their use.

1) Question No. 1 is compulsory.
2) Out of remaining questions, attempt any 4 questions.
3) **In all 5 questions to be attempted.**
4) All questions carry equal marks.
5) **Answer to each new question to be started on a fresh page.**
6) **Figures in brackets on the right hand side indicate full marks.**
7) **Assume suitable data if necessary.**

Q.1 a) **i)** Which central moments is always zero (1)
a) μ_1 b) μ_2 c) μ_3 d) None of these

ii) Let X be a continuous random variable with probability distribution (1)
$f(x) = \begin{cases} ke^{-2x}; 0 < x \\ 0 \quad ; \text{otherwise} \end{cases}$ then the value of k is
a) $-1/2$ b) $1/2$ c) $1/4$ d) None of these

iii) The joint probability mass function of X and Y is given by $p(x, y) = (2x + 3y)/72$ (1)
if $x = 1$ & $y = 2$ then the probability is
a) 5/72 b) 1/18 c) 1/9 d) None of these

iv) If mean and standard deviation of a binomial distribution is 50 and 5 respectively (1)
then trail size is
a) 20 b) 100 c) 50 d) None of these

v) Suppose the correlation coefficient between height (as measured in feet) versus weight (1)
(as measured in pounds) is 0.40. What is the correlation coefficient of height measured in inches versus weight measured in ounces? [12 inches = one foot; 16 ounces = one pound]
a) 0.30 b) 0.53 c) 0.40 d) cannot be determined from information given

vi) Two fair dice are thrown independently, then probability of 3 multiples on second dice face.
a) 16/36 b) 9/36 c) 6/36 d) 12/36 (1)

vii) Mean of gamma distribution (parameters are λ and k) is (1)

a) $k\lambda$ b) k/λ c) $k\backslash\lambda^2$ d) None of these

viii) If X and Y are independent variables then conditional function of Y given X i.e. (1)
$f(y|x)$ is

a) $f(x,y)/f_Y(y)$ b) $f_X(x)/f(x,y)$ c) $f_X(x)$ d) $f_Y(y)$

ix) Which of the following values could represent a correlation coefficient? (1)

a) e^2 b) $-4/e$ c) $\sqrt{\pi/6}$ d) $-\pi$

x) For goodness of fit, we use (1)

a) F-test b) t-test c) Z-test d) χ^2-test

b) Match the following (10)

A)	Let X be a normal variable and mean is μ, then $P(\mu \leq Z)$ is	i) $\mu \neq 12.5$
B)	If join pdf is $f(x,y) = \frac{1}{8}$; $0 < x < 2, 0 < y < 4$ then marginal probability of Y is	ii) 1/3
C)	For large sample test, if want to check that papulation mean is 12.5 or not. Then alternate hypothesis is	iii) 0.5
D)	If mean of Binomial distribution is 15 and variance is 10 then value of p is	iv) 0.25
E)	If both line of regression passes through origin then mean of x is	v) $\mu = 12.5$
		vi) 0

Q.2 (a) A lot consists total 16 articles, where 10 good article, 4 with major defects and 2 with minor defects. Two articles are chosen from the lot at random without replacement find the probability that i) both are good ii) both have major defects iii) at least one is good. (6)

(b) If X is a poisson variate such that $E(X^2) = 6$, find $E(X)$. (6)

(c) Alex, Alicia, and Juan fill orders in a fast-food restaurant. Alex incorrectly fills 20% of the orders he takes. Alicia incorrectly fills 12% of the orders she takes. Juan incorrectly fills 5% of the orders he takes. Alex fills 30% of all orders, Alicia fills 45% of all orders, and Juan fills 25% of all orders. An order has just been filled. (8)

a. Who filled the order is unknown, but the order was filled incorrectly. What is the probability that Juan filled the order?

b. Who filled the order is unknown, but the order was filled correctly. What are the probability that Alex filled the order?

Q.3 **(a)** Calculate the rank correlation coefficient from the following data. **(6)**

Height	60	62	64	66	68	70	72	74
Weight	92	83	101	110	128	119	137	146

(b) A certain drug is claimed to be effective in curing cold. In an experiment on 500 persons with cold, half of them were given the drug and half of them were given the sugar pills. The patient's reactions to the treatment are recorded in the following table: **(6)**

	Helped	Harmed	No effect	Total
Drug	150	30	70	250
Sugar pills	130	40	80	250
Total	280	70	150	500

On the basis of the data can it be concluded that there is a significant difference (at 0.05 significant level) in the effect of the drug and sugar pills?

(c) From the following information find both line of regression also find (i) x when $y = 75$ and (ii) y when $x = 30$. **(8)**

	x	y
Mean	36	85
S.D.	11	8

Correlation Coefficient=0.66

Q.4 **(a)** A fast-food chain has developed a new process to ensure that orders at the drive-through are filled correctly. The previous process filled orders correctly 85% of the time. Based on a sample of 100 orders using the new process, 94 were filled correctly. At the 0.01 level of significance, can you conclude that the new process has increased the proportion of orders filled correctly? **(6)**

(b) A man buys 50 electric bulbs of 'Philips' and 50 electric bulbs of 'HMT'. He finds that 'Philips' bulbs give an average life of 1500 hours with standard deviation of 60 hours and 'HMT' bulbs give an average life of 1512 hours with standard deviation of 80 hours. Is there a significant difference at 1% level in the mean life of bulbs? **(6)**

(c) A sample of 87 professional working women showed that the average amount paid annually into a private pension fund per person was \$3352. The population standard deviation is \$1100. A sample of 76 professional working men showed that the average amount paid annually into a private pension fund per person was \$5727, with a population standard deviation of \$1700. A women's activist group wants to "prove" that women do not pay as much per year as men into private pension funds. If they use $\alpha = .001$ and these sample data, will they be able to reject a null hypothesis that women annually pay the same as or more than men into private pension funds? **(8)**

Q.5 **(a)** Two independent samples of 8 and 7 items respectively had the following values of the variable: S I: 09, 11, 13, 11, 15, 9, 12, 14 ; S II: 10, 12, 10, 14, 9, 8, 10. **(6)**

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Is the difference between the means of the sample significant? Level of significance 5%.?

(b) Test that variance in sample 1 is small then variance in sample 2 by using the given sample information and $\alpha = .01$ Assume the populations are normally distributed. (6)

(c) A random variable X has probability density function $f(x) = 1; 0 < x < 1$ otherwise $f(x) = 0$. Find skewness and Kurtosis. (8)

Q.6 (a) The join probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y); x = 0,1,2;\ y = 1, 2, 3$. Find marginal probability of X and Y, also find $P(X + Y = 2)$. (6)

(b) The join pdf of two-dimensional RV (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8};\ 0 \leq x \leq 2\ \&\ 0 \leq y \leq 1$. Compute $P(X > 1)$ and $P\left(Y < \frac{1}{2}\right)$. (6)

(c) For certain normal distribution exactly 8% of items are below 40 and 90% of items are below 60. Find the mean and standard deviation of the distribution. Also find $P(40 < X < 60)$. (8)

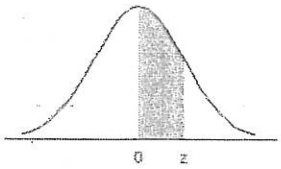
Q.7 (a) A discrete RV X takes the values -1, 0, 1 with probabilities 1/8, ¾, 1/8 respectively. Evaluate $P\{|X - \mu| \geq 2\sigma\}$ and compare it with the upper bound given by Chebyshev's inequality. (6)

(b) Find a least squares straight line for the following data: (6)

x	1	2	3	4	5
y	2	3	5	8	10

(c) Three card players play a series of matches. The probability that player A will win any game is 20%, the probability that player B will win is 30%, and the probability player C will win is 50%. If they play 6 games, what is the probability that (8)
i) player A win all 6 game ii) player B win all 6 game iii) player C win all 6 game
iv) A will win 1 game, player B will win 2 games, and player C will win 3 games

Table of Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Note:

Middle 50% area = ± 0.6745
Middle 90% area = ± 1.6449
Middle 95% area = ± 1.9600
Middle 99% area = ± 2.5758

For any Normal Variate (with Mean = μ and Std. Deviation = σ):

μ ± 1σ = 0.6827
μ ± 2σ = 0.9545
μ ± 3σ = 0.9973

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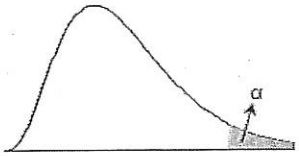
Table of Chi-Square (χ^2) Distribution



	← Level of significance (α) →									
d.f. ↓	**0.995**	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1	0.0000	0.0002	0.0010	0.0039	0.0158	2.7055	3.8415	5.0239	6.6349	7.8794
2	0.0100	0.0201	0.0506	0.1026	0.2107	4.6052	5.9915	7.3778	9.2103	10.5966
3	0.0717	0.1148	0.2158	0.3518	0.5844	6.2514	7.8147	9.3484	11.3449	12.8382
4	0.2070	0.2971	0.4844	0.7107	1.0636	7.7794	9.4877	11.1433	13.2767	14.8603
5	0.4117	0.5543	0.8312	1.1455	1.6103	9.2364	11.0705	12.8325	15.0863	16.7496
6	0.6757	0.8721	1.2373	1.6354	2.2041	10.6446	12.5916	14.4494	16.8119	18.5476
7	0.9893	1.2390	1.6899	2.1673	2.8331	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.3444	1.6465	2.1797	2.7326	3.4895	13.3616	15.5073	17.5345	20.0902	21.9550
9	1.7349	2.0879	2.7004	3.3251	4.1682	14.6837	16.9190	19.0228	21.6660	23.5894
10	2.1559	2.5582	3.2470	3.9403	4.8652	15.9872	18.3070	20.4832	23.2093	25.1882
11	2.6032	3.0535	3.8157	4.5748	5.5778	17.2750	19.6751	21.9200	24.7250	26.7568
12	3.0738	3.5706	4.4038	5.2260	6.3038	18.5493	21.0261	23.3367	26.2170	28.2995
13	3.5650	4.1069	5.0088	5.8919	7.0415	19.8119	22.3620	24.7356	27.6882	29.8195
14	4.0747	4.6604	5.6287	6.5706	7.7895	21.0641	23.6848	26.1189	29.1412	31.3193
15	4.6009	5.2293	6.2621	7.2609	8.5468	22.3071	24.9958	27.4884	30.5779	32.8013
16	5.1422	5.8122	6.9077	7.9616	9.3122	23.5418	26.2962	28.8454	31.9999	34.2672
17	5.6972	6.4078	7.5642	8.6718	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185
18	6.2648	7.0149	8.2307	9.3905	10.8649	25.9894	28.8693	31.5264	34.8053	37.1565
19	6.8440	7.6327	8.9065	10.1170	11.6509	27.2036	30.1435	32.8523	36.1909	38.5823
20	7.4338	8.2604	9.5908	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968
21	8.0337	8.8972	10.2829	11.5913	13.2396	29.6151	32.6706	35.4789	38.9322	41.4011
22	8.6427	9.5425	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7957
23	9.2604	10.1957	11.6886	13.0905	14.8480	32.0069	35.1725	38.0756	41.6384	44.1813
24	9.8862	10.8564	12.4012	13.8484	15.6587	33.1962	36.4150	39.3641	42.9798	45.5585
25	10.5197	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9279
26	11.1602	12.1981	13.8439	15.3792	17.2919	35.5632	38.8851	41.9232	45.6417	48.2899
27	11.8076	12.8785	14.5734	16.1514	18.1139	36.7412	40.1133	43.1945	46.9629	49.6449
28	12.4613	13.5647	15.3079	16.9279	18.9392	37.9159	41.3371	44.4608	48.2782	50.9934
29	13.1211	14.2565	16.0471	17.7084	19.7677	39.0875	42.5570	45.7223	49.5879	52.3356
30	13.7867	14.9535	16.7908	18.4927	20.5992	40.2560	43.7730	46.9792	50.8922	53.6720

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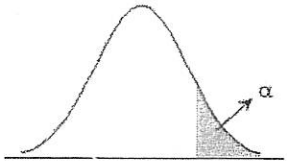
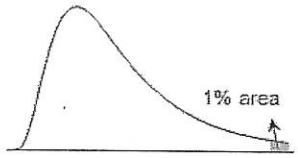
Table of 't' Distribution



	← Level of significance (α) →				
d.f. ↓	0.10	0.05	0.025	0.01	0.005
1	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.3212	1.7171	2.0739	2.5083	2.8188
23	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.3104	1.6973	2.0423	2.4573	2.7500
∞	1.2816	1.6449	1.9600	2.3264	2.5758

Table of 'F' Distribution. (Value of $F_{0.01}$)



← Degrees of Freedom for the Denominator →	← Degrees of Freedom for the Numerator →								
	1	2	3	4	5	6	7	8	9
1	4,052.18	4,999.50	5,403.35	5,624.58	5,763.65	5,858.99	5,928.36	5,981.07	6,022.47
2	98.5025	99.0000	99.1662	99.2494	99.2993	99.3326	99.3564	99.3742	99.3881
3	34.1162	30.8165	29.4567	28.7099	28.2371	27.9107	27.6717	27.4892	27.3452
4	21.1977	18.0000	16.6944	15.9770	15.5219	15.2069	14.9758	14.7989	14.6591
5	16.2582	13.2739	12.0600	11.3919	10.9670	10.6723	10.4555	10.2893	10.1578
6	13.7450	10.9248	9.7795	9.1483	8.7459	8.4661	8.2600	8.1017	7.9761
7	12.2464	9.5466	8.4513	7.8466	7.4604	7.1914	6.9928	6.8400	6.7188
8	11.2586	8.6491	7.5910	7.0061	6.6318	6.3707	6.1776	6.0289	5.9106
9	10.5614	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511
10	10.0443	7.5594	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424
11	9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315
12	9.3302	6.9266	5.9525	5.4120	5.0643	4.8206	4.6395	4.4994	4.3875
13	9.0738	6.7010	5.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911
14	8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297
15	8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948
16	8.5310	6.2262	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804
17	8.3997	6.1121	5.1850	4.6690	4.3359	4.1015	3.9267	3.7910	3.6822
18	8.2854	6.0129	5.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971
19	8.1849	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225
20	8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567
21	8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981
22	7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458
23	7.8811	5.6637	4.7649	4.2636	3.9392	3.7102	3.5390	3.4057	3.2986
24	7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560
25	7.7698	5.5680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172
30	7.5625	5.3903	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665
40	7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876
60	7.0771	4.9774	4.1259	3.6490	3.3389	3.1187	2.9530	2.8233	2.7185
120	6.8509	4.7865	3.9491	3.4795	3.1735	2.9559	2.7918	2.6629	2.5586
∞	6.6433	4.6122	3.7881	3.3254	3.0233	2.8080	2.6453	2.5172	2.4132

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