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SVKM's NMIMS
MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING/
SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING

Programme: B.Tech/ MBA Tech (Computer) Year: II Semester: III

Academic Year: 2019-20

Subject: Discrete Mathematics

Marks: 100
Date: 12 November 2019 Time: 2.00 pm - 5.00 pm
Durations: 3 (Hrs)
No. of Pages: 03

Final Examination (2019-20)

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover of the Answer Book, which is provided for their use.

1) Question No. _1___ is compulsory.
2) Out of remaining questions, attempt any __4___ questions.
3) **In all ___5_ questions to be attempted.**
4) All questions carry equal marks.
5) **Answer to each new question to be started on a fresh page.**
6) **Figures in brackets on the right hand side indicate full marks.**
7) **Assume suitable data if necessary.**

Q. 1) A) **a)** If $A \subseteq B$ and $B \subseteq C$ then $A \cup (B \cap C)$ equals to _______ [1]

b) If the universe of discourse is {1,2}, then the truth value of $(\exists x)(x^5 + x^4 = 12)$ equals to ________ [1]

c) If $A = \{1,2,3\}$ then total number of equivalence relations on A are _____ [1]

d) Letters of the word ASSASSINATION can be arranged in _____ number of ways so that all the 'S's are together. [1]
i) 151200 ii) 151212 iii) 151324 iv) 151222

e) If $a_0 = 3$ and $a_{n+1} = 2a_n + 3$ then a_1, a_2, a_3, a_4 are given by [1]
i) $a_1 = 7, a_2 = 23, a_3 = 45, a_4 = 93$
ii) $a_1 = 8, a_2 = 22, a_3 = 44, a_4 = 93$
iii) $a_1 = 9, a_2 = 21, a_3 = 46, a_4 = 94$
iv) $a_1 = 9, a_2 = 21, a_3 = 45, a_4 = 93$

f) $(2Z, .)$ is ______, where ' . ' represents usual multiplication [1]
i) Groupoid ii) Semi-Group iii) Monoid iv) Group

g) Permutation group S_3 contains ______ number of elements of order 3. [1]

h) Which of the following isbipartite? [1]
i) $K_n, n \geq 3$ ii) $K_{m,n}$ iii) C_n, n is even iv) both ii and iii

i) Chromatic number 'χ' of tree is ________ [1]

j) The statement $p \rightarrow (q \rightarrow p)$ is ______ [1]
i) contingency ii) contradiction iii) tautology

B) Match the following [10]

a) Order of element $\bar{3}$ in the group $(Z_7, \times_7)$ is	i) 3
b) Chromatic number of cycle graph C_7 is	ii) 9
c)If $A = \{1,2,3,4,5\}$ then how many elements of A satisfies $(\exists x)(x+1>5)$	iii) 6
d)If any 5 numbers are chosen among 1 to 8 then the sum of any two of them is	iv) 4
e) If $\|A\|=2$ and $\|B\|=1$ then number of relations from A to B are	v) 1

Q. 2) a) Let R be a relation on set $A = \{1,2,3,4\}$. Give examples of R, satisfying the following criterion. Justify your answers. [6]

i. R is reflexive and symmetric but not transitive.

ii. R is reflexive and transitive but not symmetric.

iii. R is an symmetric, antisymmetric but not reflexive.

b) In a class, 42% students passed in Mathematics, 45% passed in Physics, 41% passed in Chemistry, 16% passed in Mathematics and Physics, 19% passed in Physics and Chemistry, 18% passed in Chemistry and Mathematics and 15% failed in all three subjects. Find the number of students who passed in all the three subjects if there were 300 students in the class. Also find the number students passed in only one subject. [6]

c) If $f: R - \left\{\frac{7}{3}\right\} \rightarrow R - \left\{\frac{4}{3}\right\}$ defined as $f(x) = \frac{4x-5}{3x-7}$. Prove that 'f ' is bijective function, also find the rule for f^{-1}. [8]

Q. 3) a) Use mathematical induction to prove that $(11)^{n+2} + (12)^{2n+1}$ is always divisible by 133. [6]

b) Show that among any 'n+1' numbers one can find two numbers so that their difference is divisible by 'n'. Hence show that among any 12 different two digit numbers one can choose two of them so that their difference is a two digit number with identical first and second digit. [6]

c) Use Euclidean algorithm to find the greatest common divisor of integers 2002 and 2339 and hence express the GCD in the linear combination of 2002 and 2339. [8]

Q. 4) a) Test the validity of the following argument: It is wrong to refuse to hire the most qualified applicant due to irrelevant criteria. If it is wrong to refuse to hire the most qualified applicant due to irrelevant criteria, then it is wrong to refuse to hire the most qualified applicant due to the color of her skin (because skin color is irrelevant criteria). Therefore, it is wrong to refuse to hire the most qualified applicant due to the color of her skin. [6]

b) Prove that $\sqrt{2}$ is an irrational number. Also prove that $\sqrt{2} + 5$ is an irrational number. [6]

c) Using the **laws** of logic prove the following: [8]

$$\sim(p \wedge q) \rightarrow [\sim p \vee (\sim p \vee q)] \equiv \sim p \vee q$$

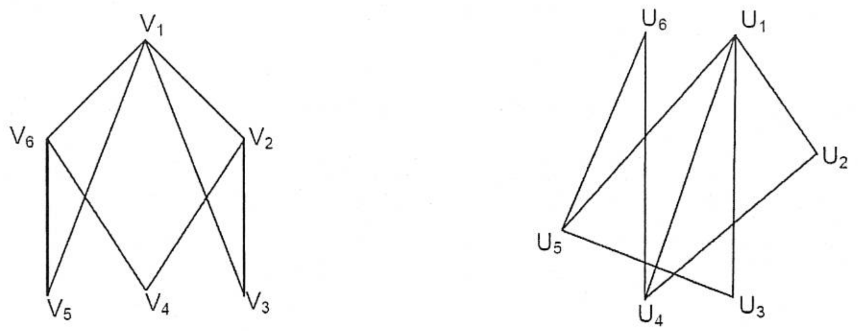
$$(p \rightarrow q) \wedge [\sim q \wedge (T \vee \sim q)] \leftrightarrow \sim (p \vee q)$$

Q. 5) a) Prove that set of congruence classes Z_6 is a commutative group under addition modulo 6 , '$+_6$' [6]

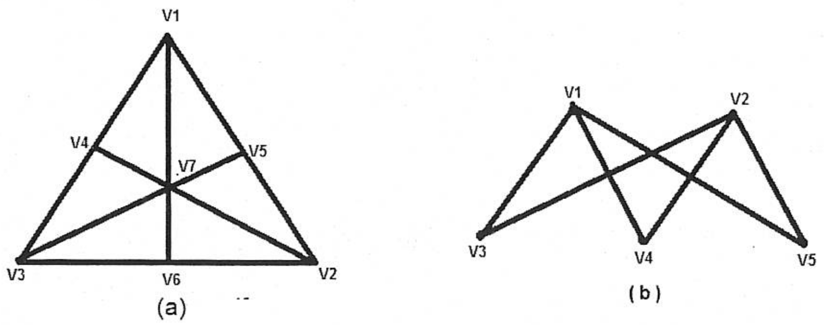
b) Prove that set of permutations 'S_3' is non-abelian group under composition of functions 'o'. [6]

c) Prove that set of integers $(\mathbb{Z}, \oplus, \otimes)$ is a commutative ring with unity under the operations defined as $a \oplus b = a + b - 1$ and $a \otimes b = a + b - ab$, where $a, b \in \mathbb{Z}$ [8]

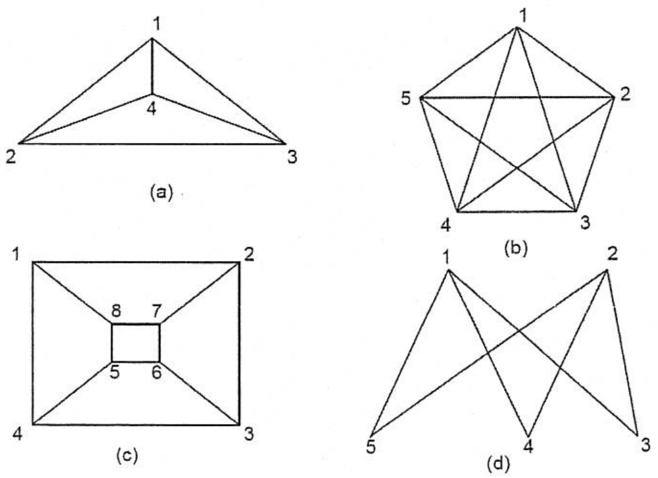
Q. 6) a) Show that the following graphs are isomorphic. **[6]**



b) Use the Welch - Powell algorithm to colour the following graphs. Also write down the corresponding chromatic number for each. **[6]**



c) Decide which of the following graphs are Eulerian or Hamiltonian or both and write down Eulerian circuit and Hamiltonian circuit wherever exists. **[8]**



Q. 7) a) If $f : (R,+) \rightarrow (R^{+},.)$ defined as $f(x) = e^{x}$, $x \in R$ then show that 'f' is group isomorphism. **[6]**

b) If G = {1, −1, i, −i} is a group under usual complex number multiplication, then prove that H = {1, −1} is a normal subgroup of G, also prove that corresponding G/H is a quotient group. **[6]**

c) Use Huffman coding to decode the following fixed length symbols with the frequencies listed: A:37, B:18, C:29, D:13, E:30, F:17, G:6. What is the average number of bits used to encode a character? **[8]**