SVKM's NMIMS

MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING/

SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING

Programme: B.Tech/ MBA Tech (All Streams)   Year: I   Semester:

Academic Year: 2019-20

Subject: Mathematics-I   Marks: 100

Date: 06 November 2019   Time: 10.00 am - 1.00 pm

                                                    Durations: 3

Hrs)

Final Examination (2019-20) / Re Examination (2018-19)

N.B.:-

1. **Question 1** compulsory

2. Attempt **any four** from the remaining questions.

3. Figures to the right indicate full marks.

COMPULSORY

Q.a i $\Gamma(n+1) = n!$ can be used when _____   2

    1. n is any integer

    2. n is a positive integer

    3. n is a negative integer

    4. n is any real number

Value of   2

$$\lim_{x \to 1} \frac{x^x - x}{x - 1 - \log x}$$

    1. 2

    2. 4

3. 0

4. 1

iii

If $x = r\cos\theta$, $y = r\sin\theta$ then $\dfrac{\partial r}{\partial x} = $

a) $\dfrac{x}{r}$ b) $\dfrac{r}{x}$ c) $\dfrac{1}{\cos\theta}$ d) none of these

iv Which of the following is the basis of subspace of the given vector space $\{(x, y, z) \in R^3 / 2x - 3y + 5z = 0\} \in R^3$

$i.[1, 1, -1]$ $ii.[1, -1, 1]$ $iii.[-1, 1, 1]$ $iv.[1, 1, 1]$

v If p=2, then rank of matrix

$$A = \begin{bmatrix} p & p & 2 \\ 2 & p & p \\ p & 2 & p \end{bmatrix}$$

is ......

1. 1

2. 2

3. 3

4. 0

1 b Match the following

The value of	$a.\dfrac{-8\sqrt{\pi}}{15}$

2/6

$\lim\limits_{x \to 0} \dfrac{\log \sin x}{\cot x}$	
The value of $\Gamma(-5/2)$	b. Grad **F** = 0
The Eigen values of A are $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$	c. (1,0,0),(0,1,0)
Let T:V_3àV_3 be linear map defined by $T(x_1,x_2,x_3) =(x_1,x_2,0)$, Range of T is	d. 0
The vector field function **F** is called solenoidal if	e. 9, 15

OPTIONAL

a Find a and b if
$$\lim\limits_{x \to 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$$
6

b Find the volume of largest possible right circular cylinder that can be inscribed in a sphere of radius a.
6

c Prove that
$$\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$$
Hence deduce that
$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{3}{25}$$
8

a If
$$z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$$

Prove that
6

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}.$$

3 b Determine a b and c such that

$$\overline{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

is irrotational.

3 c Find the equation of tangent plane and normal line to the surface at (0,0,1)

$$z = e^{-(x^2 + y^2)}$$

4 a Let T: $R^2 \to R^2$ be a linear map such that T(4,1) = (1,1), T (1,1) = (3, -2).

Compute T(1, 0)

4 b Find the range and kernel of

$$T: R^2 \to R^3$$

defined by

$$T(x, y) = (x - y, x + y, y)$$

4 c Show that the set V of positive real numbers with following operations, is a vector space.

$$x + y = xy \quad , \quad kx = x^k$$

where x and y are real numbers and k is any scalar

5 a Show that the following matrix is diagonalizable. Also find the diagonal matrix and transforming matrix,

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

5 b Reduce the matrix to row-echelon form and find its rank

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

5c

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix},$$

8

Find the characteristic equation of the symmetric matrix A hence

evaluate the matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

a   Determine the volume of solid generated by revolving the area enclosed by the   6

loop of the curve $y^4 = x(4-x)$ about the x- axis

6 b

$$\int_{-\pi/4}^{\pi/4} (\sin\theta + \cos\theta)^{1/3} d\theta$$

6

Evaluate

c

$$\int_{2}^{\infty} \frac{1}{(x-1)(x^2+1)} dx$$

8

Evaluate

7 a   Find a point on the plane $x+2y+3z = 13$ nearest to the point (1, 1, 1) using   6

the method of Lagrange multipliers.

b Verify Rank- nullity theorem for the linear transformation   6

$T: M_{22} \to M_{22}$

defined by

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a-b & 0 \\ 0 & c-d \end{pmatrix}$$

c Find the characteristic equation of the matrix A given below and hence it's inverse & the   8

matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

where

5/6

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$