51

SVKM's NMIMS

MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING

Programme: B. Tech (All Streams) Year: I Semester: I

Academic Year: 2019-20

Subject: Engineering Mathematics - I

Marks: 70

Date: 06 November 2019

Time: 10.00 am - 1.00 pm

Durations: 3 (hrs)

No. of Pages: 02

Re-Examination (2016-17/ 2017-18)

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover of the Answer Book, which is provided for their use.

1) Question No. 1 is compulsory.
2) Out of remaining questions, attempt any 4 questions.
3) **In all 5 questions to be attempted.**
4) All questions carry equal marks.
5) **Answer to each new question to be started on a fresh page.**
6) **Figures in brackets on the right hand side indicate full marks.**
7) **Assume suitable data if necessary.**

Q.1 (a) Find the general value of $\log(1+i)+\log(1-i)$. (4M)

(b) Verify Rolle's Theorem for $f(x) = x^2;\ [-1,1]$. (3M)

(c) If $u = \log\left[\frac{x^3+y^3}{x^2+y^2}\right]$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$. (4M)

(d) Find the unit vector tangent to the space curve $x = t, y = t^2, z = t^3$ at $t = 1$. (3M)

Q.2 (a) Express $\sin(6\theta)$ in terms of powers of $\sin\theta$ and $\cos\theta$. (4M)

(b) If $5\sinh x - \cosh x = 5$, find $\tanh x$. (5M)

(c) If $2\cosh\left(\alpha + i\frac{\pi}{4}\right) = x + iy$, prove that $x^2 - y^2 = 2$. (5M)

Q.3 (a) Expand $\sqrt{x}$ using Taylor's theorem, hence find $\sqrt{25.15}$. (5M)

(b) Verify LMVT for $f(x) = \log_e x$ in $[1, e]$. (5M)

(c) Evaluate $\lim_{x\to1}(1-x)\cdot\tan\left(\frac{\pi x}{2}\right)$. (4M)

Q.4 (a) If $u = 1/r, r = \sqrt{x^2+y^2+z^2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$. (5M)

(b) Find the maxima and minima of $f = x^3 + y^3 - 3axy$; where $a > 0$. (5M)

(c) If $u = x^3\sin^{-1}\left(\frac{y}{x}\right) + x^4\tan^{-1}\left(\frac{y}{x}\right)$, find the value of (4M)

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}.$$

(5M)

Q.5

(a) If $z = \phi(x,y); x = uv,\ y = u/v$, prove that $\frac{\partial z}{\partial x} = \frac{1}{2v}\cdot\frac{\partial z}{\partial u} + \frac{1}{2u}\cdot\frac{\partial z}{\partial v}$ and $\frac{\partial z}{\partial y} = \frac{v}{2}\cdot\frac{\partial z}{\partial u} - \frac{v^2}{2u}\cdot\frac{\partial z}{\partial v}$. (5M)

(b) If $u = \sin^{-1}\sqrt{x^2 + y^2}$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. (4M)

(c) Estimate the maximum error in $f(x,y) = x^2 + y^2 + xy$ at the point $x = 2,\ y = 3$ if maximum errors ± 0.01 and ± 0.02 are made in x and y respectively. (5M)

Q.6

(a) Determine the constants a, b, c so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. Hence find the scalar potential $\emptyset$ s.t. $\vec{F} = grad\phi$. (5M)

(b) If $\vec{F} = x^2\hat{\imath} + xz\hat{\jmath} + yz\hat{k}$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, find $div(\vec{F}\times\vec{r})$ and $curl(\vec{F}\times\vec{r})$. (4M)

(c) Find the values of the constants a, b, c so that the D.D. of $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, -1) has a maximum magnitude 64 in the direction parallel to z-axis.

Q.7

(a) Prove that $\text{sech}^{-1}(\sin\theta) = \log(\cot\theta/2)$. (4M)

(b) Find the principal value of $(1 + i)^{1-i}$. (5M)

(c) Prove that $\nabla f(r) = f'(r)\left(\frac{\vec{r}}{r}\right)$ and hence, find f if $\nabla f = 2r^4\vec{r}$. (5M)
